U.G. 6th Semester Examination - 2020

MATHEMATICS

Course Code : BMTMDSRT-3 & 4 (DSE 3 & 4)

Full Marks : 40 Time : 2 Hours

The figures in the right-hand margin indicate marks.

Candidates are required to give their answers in their own words as far as practicable.

Notations and symbols have their usual meanings.

This question papers contains both DSE 3 & 4. Students are thereby instructed to answer DSE paper out of these two (DSE 3 & DSE 4) as he/she opted for.

Title : Probability and Statistics

Code : BMTMDSRT3 (DSE 3)

1. Answer any **ten** questions: $1\times10=10$

a) A coin is tossed 3 times. Write down the sample space.

b) What is a Bernoulli trial?

c) Find the mean of the first n natural numbers.

d) Define probability distribution function for a discrete random variable, having spectrum $\{x_1, x_2, x_3, ...\}$.

e) Consider the function given by

$$f(x)=\begin{cases} 6x(1-x), & \text{if } 0\le x\le 1 \\ 0, & \text{elsewhere}. \end{cases}$$

Is f(x) define a probability density function?

f) When a statistic T is said to be an unbiased estimator of a parameter θ?

g) A population consists of the four members 3, 7, 11, 15. Find the population mean.

h) State Chebyshev's inequality.

i) If X has a binomial $B\left(10, \frac{1}{2}\right)$ distribution, write down the probability mass function.

j) True or False: If X and Y are independent then $Cov(X, Y)=0$.

k) Define mathematical expectation for a continuous random variable.

l) For what value of a will the function f(x)=ax, x=1, 2, 3, ..., n; be the probability mass function of a discrete random variable X?

m) Write down the density curve of the normal $N(m, \sigma^2)$ distribution.

n) What do you mean by a standard normal variate?

o) Define moment generating function for a random variable X.

2. Answer any **five** questions: 2×5=10

a) For a frequency distribution, show that

$$|\text{Mean} - \text{Median}| \le \text{Standard deviation}.$$

b) If X and Y are two random variables, then show that $\text{Cov}(X, Y) = E(XY) - E(X)E(Y)$.

c) If X is a discrete random varible having probability mass function:

x	0	1	2	3	4	5	6	7
P(X=x)	0	k	2k	2k	3k	k^2	$2k^2$	$7k^2+k$

Determine the constant k.

d) Suppose X be a Poisson μ variate, find the moment generating function of X.

e) If the joint distribution of X and Y be given by the probability density function

$$f(x, y) = \begin{cases} x + y, & \text{if } 0 < x < 1, 0 < y < 1 \\ 0, & \text{otherwise} \end{cases}$$

then find E(XY).

f) What do you know about a Scatter Diagram?

g) Define a bivariate normal distribution.

h) Show that the correlation coefficient is the geometric mean between the regression coefficients.

3. Answer any **two** questions: 5×2=10

a) i) The probability density function of a two dimensional random variable (X, Y) is

$$f(x, y) = \begin{cases} \frac{1}{8}(x + y), & \text{if } 0 \le x \le 2, \ 0 \le y \le 2 \\ 0, & \text{elsewhere}. \end{cases}$$

Find the marginal density functions $f_X(x)$ and $f_Y(y)$.

ii) If X be a random variable and a, b are real numbers, then show that

$$E(aX + b) = a\,E(X) + b. \qquad 3+2=5$$

b) i) Let $X_1, X_2, ..., X_n$ be a random sample from a population. Define sample mean and sample variance.

ii) If two random variables X and Y are independent, then prove that they are uncorrelated. Define conditional expectation for a joint continuous random variable (X, Y). 2+(2+1)=5

c) Consider the probability density function $f(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-m)^2}{2\sigma^2}}, \ (\sigma > 0)$ of a normal

$N(m, \sigma^2)$ distribution. Prove that

$\int_{-\infty}^{\infty} f(x)\, dx = 1$. 5

4. Answer any **one** question: $10\times1=10$

a) i) Let the continuous random variable X be uniformly distributed in the interval (a, b), $-\infty < a < b < \infty$, where the probability density function is given by

$$f(x) = \begin{cases} \frac{1}{b-a}, & \text{if } a < x < b \\ 0, & \text{elsewhere.} \end{cases}$$

Show that $E(X) = \frac{b+a}{2}$ and $Var(X) = \frac{(b-a)^2}{12}$.

ii) The probability density function of a continuous bivariate distribution is given by the joint density function

$$f(x, y) = \begin{cases} x+y, & \text{for } 0 < x < 1,\ 0 < y < 1 \\ 0, & \text{otherwise.} \end{cases}$$

Find the correlation coefficient $\rho(X, Y)$. 4+6=10

b) i) If $X_1, X_2, ..., X_n$ is a random sample from an infinite population with variance σ^2 and $\bar{X}$ is the sample mean, show that $\sum_{i=1}^{n} \frac{1}{n}(X_i - \bar{X})^2$ is a biased estimator of σ^2.

ii) Let T_1 and T_2 be two unbiased estimator of the parameter θ. Under what condition aT_1+bT_2 will be an unbiased estimator?

iii) If X and Y are two random variables such that $X \le Y$, then prove that $E(X) \le E(Y)$. 5+3+2=10

c) i) If the correlation coefficient between the random variables X and Y is $\frac{1}{2}$, then find the correlation coefficient between the random variables U=5X and V=–3Y.

ii) The joint probability density function of X and Y is

$$f(x, y) = \begin{cases} 8xy, & \text{if } 0 \le x \le 1,\ 0 \le y \le 1 \\ 0, & \text{elsewhere.} \end{cases}$$

Examine where X and Y are independent.

iii) If X and Y are two random variables, show that

$$Var(X+Y) = Var(X) + Var(Y) + 2Cov(X, Y).$$

4+3+3=10

Title : Mechanics-II

Code : BMTMDSRT4 (DSE 4)

1. Answer any **ten** questions: 1×10=10

a) Define virtual work.

b) What is a couple?

c) What do you understand by a common catenary?

d) State stable equilibrium of a body.

e) What are the conditions that a given system of forces may be reduced to a single resultant force acting on a rigid body?

f) What is deformable body?

g) Define an equi-pressure surface in a fluid.

h) What is meant by the stress component T_{xy} at a point (x, y, z) in a continuum medium?

i) Write down stress matrix for a perfect fluid.

j) What is Archimedes' principle?

k) Write down the dimension of Pressure-gradient and Normal stress.

l) Define pressure at a point in a fluid.

m) When a fluid is said to be non-homogeneous?

n) What is a perfect fluid?

o) Write down the differential equation of a fluid in equilibrium.

2. Answer any **five** questions: 2×5=10

a) Define Poinsot's central axis.

b) What are the invariants of a given system of forces acting on a rigid body about any base point?

c) State the principle of virtual work for any system of co-planar forces acting on a rigid body.

d) What is the centre of pressure for a surface immersed in a liquid? Is it a single point? Give reasons.

e) Define body force and surface force with examples.

f) Express pressure derivative in terms of external force of a fluid in equilibrium.

g) What is an isothermal process and an adiabatic process?

h) Show that the free surface of a homogeneous liquid at rest under gravity is horizontal.

3. Answer any **two** questions: 5×2=10

a) i) State and verify the conditions of equilibrium of a system of forces acting at different points on a body.

ii) Show that a force and a couple can not produce equilibrium. 3+2

b) Let $P(x, y, z)$ be any point on a fluid in equilibrium and X, Y, Z be the components of external force per unit mass parallel to the co-ordinate axes respectively. Prove that

$$X\left(\frac{\partial Y}{\partial z}-\frac{\partial Z}{\partial y}\right)+Y\left(\frac{\partial Z}{\partial x}-\frac{\partial X}{\partial z}\right)+Z\left(\frac{\partial X}{\partial y}-\frac{\partial Y}{\partial x}\right)=0.$$

5

c) A semi-circular lamina of radius a is immersed vertically in a liquid, the pressure on which varies as the depth; with the bounding diameter in the surface. Find the centre of pressure of the lamina. 5

4. Answer any **one** question: 10×1=10

a) i) If p be the pressure, ρ be the density and $\vec{F}=(X, Y, Z)$ be the external force per unit mass at a point (x, y, z) of a fluid in equilibrium, then show that $dp=\rho(Xdx+Ydy+Zdz)$.

ii) Show that the system of forces per unit mass given by $X=\lambda y(a^2+z^2)$, $Y=-\lambda x(a^2+z^2)$, $Z=\mu(x^2+y^2)$, where λ, μ, a are constants, can keep a fluid in equilibrium.

iii) State Archimedes' principle for a freely floating body. 4+4+2

b) i) A liquid fills the half of a circular tube of radius 'a' in a vertical plane. If the tube is now rotated about the vertical diameter with uniform angular velocity ω such that the liquid is about to separate in two parts, show that $\omega=\sqrt{2g/a}$.

ii) A hemispherical bowl is filled with liquid and placed in an inverted position in contact with a horizontal table and no water comes out. Show that the resultant vertical thrust on its curved surface is one-third of the thrust on the table.

5+5

c) i) Show that a given system of forces can have only one central axis.

ii) Define Wrench of a system of forces acting on a rigid body.

iii) Six forces, each equal to P, act along the edge of a cube, taken in order, which do not meet a given diagonal. Show that their resultant is a couple of moment $2\sqrt{3}\,Pa$, where a is the edge of the cube.

4+2+4

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